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DOES EFFICIENCY OF HIGH ENERGY COLLISIONS DEPEND ON A HARD SCALE?

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ABSTRACT

The multiplicity of charged hadrons in the current fragmentation region of both the c.m.s. and the Breit frame of deep inelastic scattering is calculated and compared with the HERA data. The results are in agreeement with Yang's hypothesis that the efficiency of high energy processes increases at larger momentum transfer.

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1 Introduction

It seems quite natural to expect that the harder a high-energy collision is, the higher is the number of fragments. One of the most tractable and widely explored processes where this phenomenon can be seen is deeply inelastic scattering (DIS) with a possibility to change a hard scale (Q^2, gauge boson virtuality) and to detect its influence (if any) on the hadronic invariant mass (W) "efficiency". The simplest measure of this efficiency is the multiplicity of secondaries.

In 1969 Yang and his collaborators [1], basing themselves on the "fragmentation picture" of violent collisions, made a qualitative prediction: "...for larger values of the momentum transfer t, the breakup process favors larger multiplicities of hadrons" (at fixed hadronic mass). Early searches for this effect were inconclusive in both theory and experiment [2].

In the framework of QCD a quantitative result has been obtained in Refs. [3, 4]: it appears that QCD gluon bremsstrahlung leads to an increase of the hadron multiplicities in DIS, with an increase of Q^2 at fixed hadronic mass W, but that this increase is very slow. The distinctive feature of this result is that $\langle n \rangle^{DIS}(W, Q^2)$ has a finite limit at $Q^2 \to \infty$ and W fixed.

Later, another result has been claimed in Ref. [5], which predicted an infinite and quite rapid growth of $\langle n \rangle^{DIS}(W, Q^2)$ with Q^2. In the course of the inference of this result it was supposed that the influence of the (non-perturbative) composite structure of the nucleon is negligible while in [3, 4] it plays a key role in the slowness of the Q^2 dependence of the multiplicity at fixed W. There is even more trivial objection. On general grounds, the infinite growth with Q^2 at fixed W is impossible because of the *apriori* kinematical bound

$$\langle n \rangle^{DIS}(W, Q^2) \leq \frac{W}{m}, \tag{1}$$

where m is some effective mass.

Quite recently, a weak dependence on Q^2 in the framework of the Dual Parton Model was mentioned in [6].

Experimentally a statistically significant effect of the slow growth of $\langle n \rangle^{DIS}(W, Q^2)$ was established in $\nu(\bar{\nu})p$ interactions [7] and in $\mu^+ p$ interactions [8]. The results of the EMC [8] have been described in the framework of QCD in Ref. [9].

However, subsequent measurements at HERA (H1) were interpreted as a practical Q^2 independence [10] of $\langle n \rangle^{DIS}(W, Q^2)$ (for the current hemisphere in the hadronic c.m.s.), while H1 [12, 13] and ZEUS [14, 15] reported quite fast Q^2 dependence for the current hemisphere in the Breit frame. It should be noted, however, that this last result concerns different bins in W for changing Q^2 values. Anyway, the situation is controversial and therefore very interesting.

In this paper we give our own interpretation of the HERA data on charged hadron multiplicities in the current fragmentation region; as will be seen in the text below, these are in agreement with Yang's general hypothesis [1] and our early QCD results [3, 4] (see also the review [11]).

2   Hadronic Spectrum and Multiplicity in DIS

According to the factorization for inclusive spectra in DIS, the hadronic spectrum in DIS is represented by two terms:

$$\frac{dn}{dy}^{DIS}(W, Q^2) = \int_{x_0}^1 \frac{dz}{z} w(x, z, Q^2) \frac{d\hat{n}}{dy}(W_{eff}, Q^2) + \frac{dn_0}{dy}, \tag{2}$$

where $x_0 = x + (1 - x)(m_h/W)\exp(-y)$, y is the rapidity of the detected hadron and m_h is its mass. In Eq. (2) $d\hat{n}/dy$ defines the hadronic spectrum in partonic subprocess, while the quantity dn_0/dy describes the spectrum of the proton remnant. The latter does not contribute to the current fragmentation region of DIS at HERA energies.

Correspondingly, the average hadronic multiplicity in DIS is represented by

$$\langle n \rangle^{DIS}(W, Q^2) = \int_{x_0}^1 \frac{dz}{z} w(x, z, Q^2) \langle \hat{n} \rangle (W_{eff}, Q^2) + \langle n_0 \rangle. \tag{3}$$

For small x the weight $w(x, z, Q^2)$ in (2), (3) is of the form:

$$
\begin{aligned}
w(x, z, Q^2) &= D_g^q\left(\frac{x}{z}, Q^2, Q_0^2\right) f_g(z, Q_0^2) \\
&\times \left(\int_{x_0}^1 \frac{dz}{z} D_g^q\left(\frac{x}{z}, Q^2, Q_0^2\right) f_g(z, Q_0^2)\right)^{-1}.
\end{aligned}
\tag{4}
$$

As can be seen, the hadronic spectrum in partonic subprocess, $d\hat{n}/dy$, and the hadronic multiplicity $\langle \hat{n} \rangle$ depend on the effective energy, which is smaller than W:

$$W_{eff}^2 = \frac{z - x}{1 - x} W^2. \tag{5}$$

In what follows, we shall work in the c.m.s. of the final hadrons. In terms of rapidity, the current region in the c.m.s. corresponds to

$$-Y < y < 0 \tag{6}$$

(it is assumed that the proton goes in the positive direction).

In our papers [3, 4] it has been established that the total hadronic multiplicity in the partonic subprocess of DIS is related to the hadronic multiplicity in $e^+ e^-$ annihilation:

$$\langle \hat{n} \rangle (W, Q^2) \simeq \langle n \rangle^{e^+ e^-}(W) \tag{7}$$

(up to small NLO corrections, which decrease in Q^2).

In the partonic subprocess, the rapidity varies in the range

$$-\hat{Y} < y + y_0 < \hat{Y}, \tag{8}$$

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where $\hat{Y} = \ln(W_{eff}/m_h)$ and

$$y_0 = \frac{1}{2} \ln \left(\frac{1-x}{1-z} \right). \tag{9}$$

The quantity y_0 determines the rapidity of the centre of mass of the partonic subprocess in the centre of mass of the complete process. On integration over z, the region (8) is "smeared" into the region

$$-Y < y < Y, \tag{10}$$

with $Y = \ln(W/m_h)$.

The average value of the effective energy in (2), (3), available for particle production, appeared to be dependent on both W and Q^2 [3, 4]:

$$\langle W_{eff} \rangle^2 \simeq \kappa(Q^2)W^2. \tag{11}$$

The efficiency factor $\kappa(Q^2)$, which stands in front of W^2 in (11), is much less than 1 and grows slowly in Q^2.

From formulas (3), (7) and (11), one can see that the rise of the average hadronic multiplicity in DIS has the same physical nature as in e^+e^- annihilation. For the first time this behaviour has been experimentally established by H1 in 1996 [10].

However, the QCD growth of $\langle \hat{n} \rangle$ is delayed in DIS by the bound-state effects and the slow QCD evolution of the structure function. This is why we predicted that the Q^2 dependence of $\langle \hat{n} \rangle$ at fixed W should remain numerically weak at HERA energies [9, 11].

It follows from (8), (9) that the centre of the spectrum is shifted to the region of positive rapidities and tends to zero at asymptotically high Q^2 [4]:

$$\langle y_0 \rangle \Big|_{Q^2 \to \infty} \sim \frac{1}{\ln(\ln Q^2))}. \tag{12}$$

The hadronic spectrum in partonic subprocess in the c.m.s. of DIS has the form

$$\frac{d\hat{n}^h}{dy} = n^{e^+e^-}(W_{eff}, Q^2) \, \bar{D}^h(W_{eff}, y). \tag{13}$$

Normalization in the RHS of Eq. (13) is done in agreement with formula (3).

3   Hadronic Multiplicities in the Current Fragmentation Region

To calculate the multiplicity of charged hadrons in the current fragmentation region, we have to define expressions of the quark distribution at small x, of the hadronic spectrum in the partonic subprocess as well as of the multiplicity of charged hadrons in e^+e^- annihilation.

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For the quark distribution, we use an analytical expression from Ref. [16], in the case of soft initial conditions. As was shown in [16], at small x it is in good agreement with the data on the structure function from HERA in the wide ranges of Q^2. Namely, at high Q^2, we have

$$D_g^q(z, Q^2) \sim r I_1(t) \exp(-d\xi/2), \tag{14}$$

with $d = \beta_0 + 20 N_f/27$. The variable

$$t = 2\sqrt{6\xi \ln\left(\frac{1}{z}\right)} \tag{15}$$

is related to the QCD evolution parameter

$$\xi = \frac{2}{\beta_0} \ln\left(\frac{\alpha(Q_0^2)}{\alpha(Q^2)}\right), \tag{16}$$

where $\beta_0 = 11 - 2N_f/3$ is the β-function in lowest order and

$$r = \frac{t}{2\ln(1/z)}. \tag{17}$$

The quark and gluon distributions from Ref. [16] obey the GLAP evolution equations [17].

The expression of the initial gluon distribution at z closed to 1 is chosen to have the following form

$$f_g(z, Q_0^2)\,|_{z\to 1} \sim (1 - z)^{n_g}. \tag{18}$$

We have omitted constant factors in the RHS of Eqs. (14) and (18) as they do not influence our final results.

The spectrum of hadrons in the partonic process $\bar{D}^h$ was calculated by many authors. We use the expression from Refs. [18] (N is a normalization factor):

$$\bar{D}^h(W, \zeta) = \frac{N}{\sigma\sqrt{2\pi}} \exp\left[\frac{1}{8}k - \frac{1}{2}s\delta - \frac{1}{4}(2+k)\delta^2 + \frac{1}{6}s\delta^3 + \frac{1}{24}k\delta^4\right] \tag{19}$$

calculated in the variable

$$\zeta = \ln\left(\frac{W}{E_h}\right). \tag{20}$$

Here E_h is the energy of the detected hadron.

The average value of ζ, ζ_0, and its dispersion σ are given by the formulas:

$$\zeta_0 = \frac{1}{2}\tau\left(1 + \frac{\rho}{24}\sqrt{\frac{48}{\beta_0\tau}}\right)\left(1 - \frac{\omega}{6\tau}\right), \tag{21}$$

$$\sigma = \sqrt{\frac{\tau}{3}}\left(\frac{\beta_0\tau}{48}\right)^{1/4}\left(1 - \frac{\beta_0}{64}\sqrt{\frac{48}{\beta_0\tau}}\right)\left(1 + \frac{\omega}{8\tau}\right), \tag{22}$$

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where

$$\tau = \ln\left(\frac{W}{\Lambda}\right) \tag{23}$$

and

$$s = -\frac{\rho}{16}\sqrt{\frac{3}{\tau}}\left(\frac{48}{\beta_0\tau}\right)^{1/4}\left(1 + \frac{\omega}{4\tau}\right), \tag{24}$$

$$k = -\frac{27}{5\tau}\left(\sqrt{\frac{\beta_0\tau}{48}} - \frac{\beta_0}{24}\right)\left(1 + \frac{5\omega}{12\tau}\right), \tag{25}$$

$$\delta = \frac{\zeta - \zeta_0}{\sigma}. \tag{26}$$

Here $\rho = 11 + 2N_f/27$, $\omega = 1 + N_f/27$.

At low (effective) energies we use the fit of the low-energy data on multiplicity of charged hadrons in e^+e^- annihilation from Ref. [19]:

$$\langle n \rangle^{e^+e^-} = 2.67 + 0.48\ln W^2, \tag{27}$$

while for high energies ($W_{eff} > 10$ GeV) we apply the fit from Ref. [20], which well decsribes e^+e^- data up to LEP energies:

$$\langle n \rangle^{e^+e^-} = -1.66 + 0.866\exp(1.047\sqrt{\ln W^2}). \tag{28}$$

We have corrected (27) for a fraction of the charged particles from K_s^0 and Λ ($\bar{\Lambda}$) decays.

Figure 1 represents the result of calculations of charged multiplicity in current hemisphere of the c.m.s. by using formulas (2) and (14) (solid curves) in comparison with the H1 data from Ref. [10]. As can be seen, our QCD predictions are in very good agreement with the data. These are quite compatible with a slow growth of $\langle n \rangle^{DIS}(W, Q^2)$ in Q^2 at fixed W.

Figure 2 demonstrates a rapid rise of $\langle n \rangle^{DIS}(W, Q^2)$ in the variable W for different values of Q^2, which was predicted many years ago in Refs. [3, 4] and seen previously in e^+e^- annihilation (the very values of Q^2 taken from [10]). Let us note that the H1 data presented in Fig. 2 (see Table 4 in [10]) do not correspond to some fixed values of Q^2, in contrast with the experimental points in Fig. 1.

In order to obtain multiplicity of charged hadrons in current region of the Breit frame, the c.m.s. spectrum (13) must be integrated in the region

$$- Y < y < y_B, \tag{29}$$

where

$$y_B = -\frac{1}{2}\ln\left(\frac{1 + v}{1 - v}\right) \simeq -\frac{1}{2}\ln\left(\frac{1}{x}\right). \tag{30}$$

The quantity

$$v = \sqrt{1 - 4x(1 - x)} \tag{31}$$

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in the RHS of Eq. (30) is the velocity of the Breit frame in the c.m.s. So, y_B corresponds to zero rapidity in this frame.

The results of our calculations of the multiplicity of charged hadrons in the current region of the Breit frame are presented in Fig. 3 as a function of Q^2, in comparison with the H1 data [13] (solid squares) and ZEUS data [15] (solid circles).

The theoretical curves in Figs. 1–3 correspond to the following values of parameters:

$$Q_0^2 = 1 \text{ GeV}^2, \quad \Lambda = 0.25 \text{ GeV}. \tag{32}$$

The parameter $n_g = 6.1$ in Eq. (19) is taken from one of the MRST sets of parton distributions [21].

It should be noted that the strong Q^2 dependence seen by H1 and ZEUS in the Breit frame has nothing to do with the Q^2 dependence of $\langle n \rangle^{DIS}(W, Q^2)$ in the c.m.s. (see Fig. 1); to a large extent it has a kinematical origin. The point is that an increase of Q^2 at fixed W is equivalent to an increase of x. As a result, the current region of the Breit frame (29) enlarges. Thus, the rapid growth of hadronic multiplicity in the Breit frame in Q^2 (at fixed W) reflects a strong increase of the hadronic spectrum towards the central region.

As for the increase of $\langle n \rangle^{DIS}(x, Q^2)$ in Q^2 at fixed x in the Breit frame, it has been found that it is similar to that in e^+e^- annihilation at high Q^2, while there is a discrepancy between DIS and e^+e^- data at low Q^2 [12]–[15]. Within our approach, it can be understood as follows. On the one hand, the increase of Q^2 results in an increase of the height of the spectrum (because W grows). On the other hand, the position of the spectrum, $\langle y_0 \rangle$ as defined in (9), tends towards the region of positive rapidities.

These two phenomena go in opposite directions. At Q^2 high enough, $\langle y_0 \rangle$ varies very slowly with Q^2 (12). As a result, the rise of hadronic multiplicity in the Breit frame is analogous to that in e^+e^- annihilation. At low Q^2, $\langle y_0 \rangle$ changes more significantly [4]. This effect partially compensates the growth of the spectrum and there appears a significant difference between DIS and e^+e^- data.

Finally, it is interesting to analyse the case when x increases while Q^2 remains fixed. In this, W decreases, which results in a rapid decrease of the spectrum. At the same time, however, the current region in the Breit frame becomes larger in accordance with formulas (29), (30). The effects are of the same order but opposite in sign. The ZEUS data in the Breit frame (see Table 2 in Ref. [15]) show that there is a slow rise of the hadronic multiplicity in the current hemisphere in the variable x at different fixed values of Q^2.

4 Conclusions

In this paper we have presented the results of QCD calculations of the multiplicity of charged hadrons in the current hemisphere of DIS. Both the c.m.s. and the Breit frame are considered.

We have shown that the H1 data are in agreement with Yang's hypothesis and our QCD predictions. Namely, the efficiency of high energy collisions does weakly depend on the hard scale of the process (momentum transfer Q^2). It means that *at fixed energy* the efficiency of a particle production in hard processes increases with the shrinking of the interaction region ($\sim 1/Q$, in DIS).

The observed rise of the hadronic multiplicity with Q^2 in the current region of the Breit frame has both a dynamical and kinematical origin. This is why a direct comparison of available DIS data in the Breit frame with e^+e^- data is not completely correct.

New data from HERA on the hadronic multiplicity in the current region of the c.m.s. as well as measurements of the total multiplicity in DIS as functions of two variables (Q^2 and W/x) would be very important.

Acknowledgements

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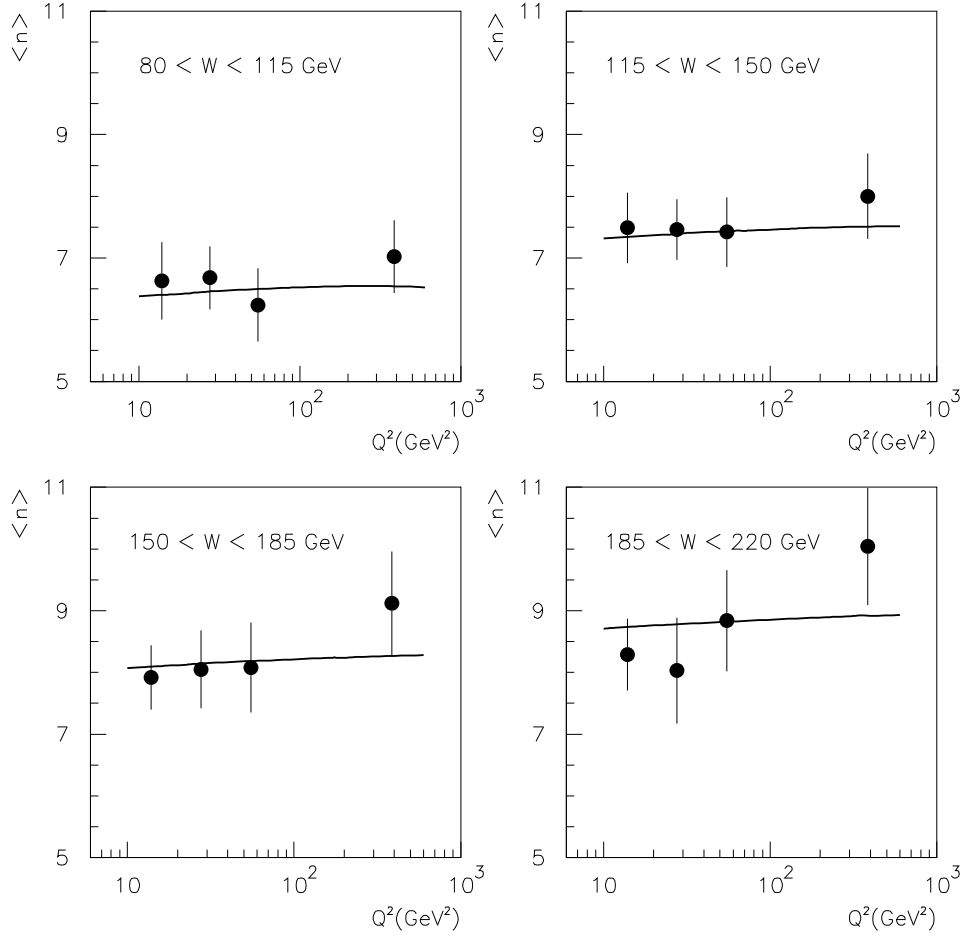[21] A.D. Martin, R.G. Roberts, W.J. Stirling and R. Thorne, Eur. Phys. J. C4 (1998) 463.

Figure 1: The Q^2 dependence of the multiplicity of charged hadrons in the current fragmentation region of the c.m.s. in intervals of W
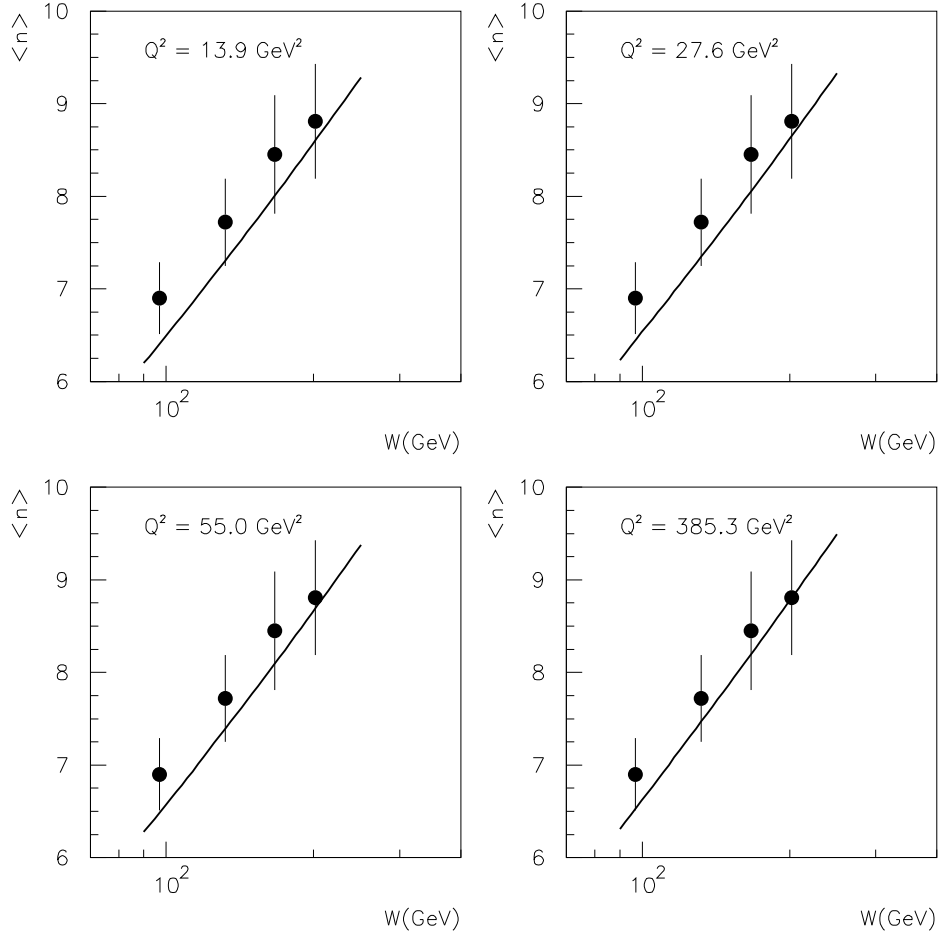
Figure 2: The W dependence of the multiplicity of charged hadrons in the current fragmentation region of the c.m.s. at fixed values of Q^2
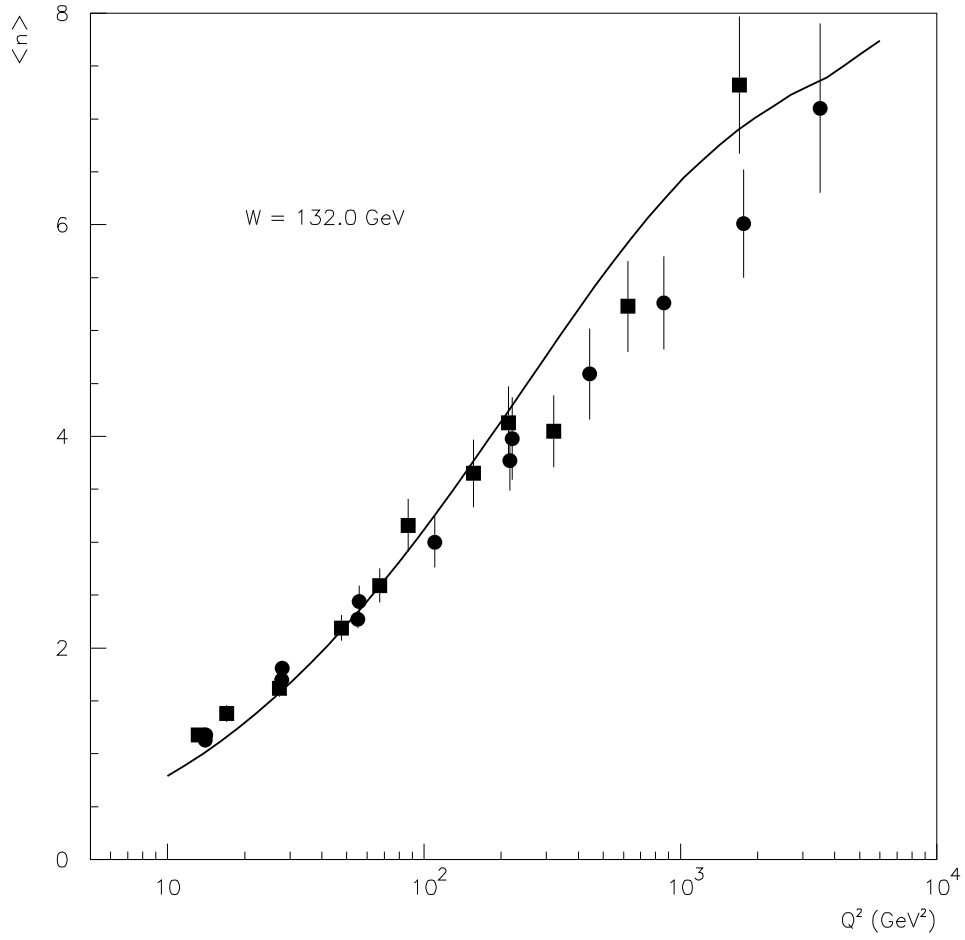
Figure 3: The Q^2 dependence of the multiplicity of charged hadrons in the current fragmentation region of the Breit frame at fixed values of W